Incorporating uncertainty in optimal investment decisions

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Abstract: Investment decisions are now more crucial than ever. The investors are in need of sound arguments, which will be able to shape the investment specifications and appraise their uncertain nature. This paper proposes an innovative approach that merges optimisation and risk analysis in one single method. The two-step investment appraisal approach reaches an optimum through a Genetic Algorithm optimisation and then assesses the environment's risk through a Monte Carlo simulation. The approach, thus, offers the best investment characteristics, as well as information about its implied risk. The use of the method is illustrated through an extensive Case Study.

Keywords: investment appraisal; genetic algorithms; GA; Monte Carlo Simulation; MCS; risk analysis; project management.

Reference to this paper should be made as follows: Rentizelas, A., Tziralis, G. and Kirytopoulos, K. (2007) 'Incorporating uncertainty in optimal investment decisions', *World Review of Entrepreneurship, Management and Sustainable Development*, Vol. 3, Nos. 3/4, pp.273–283.

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1 Introduction

The constantly increasing complexity of business ecosystems has rapidly expanded the variety, as well as the importance and risk, of potential investments. Investment decisions are now more crucial than ever for any kind of production companies, since the right investment in a global economy framework may aid the prosperity of the company, while the wrong investment may lead even to bankruptcy.

The investment's problem is as old as the economy itself. In order to succeed in optimum utilisation of the limited available investment resources, each decision should take into consideration a huge set of factors, which are either defined by the investor, or can be affected only by external factors. The decision to proceed or not to an investment is most often based upon the outcome of the dominant economic criteria, such as the Net Present Value (NPV), the Internal Rate of Return (IRR) and the Payback period (van Groenendaal and Kleijnen, 2002; Biezma and San Cristobal, 2006).

The typical investment analysis forces the decision maker to assign values to every single variable of the investment model and to perform, at best, a sensitivity analysis afterwards. This approach will lead to a non-optimal outcome, not capable of giving the highest possible amount of information to the decision maker. An alternative approach is to take into account risk management principles while estimating the outcome of an investment (Pike and Neale, 1993).

Risk management has always being regarded in the economic field as a potential fluctuation of the economic variables (Alexander, 1999). The economic and technical risks (Hammer, 1972) are the first structured approaches met in the scientific literature. However, there are many more scientific sectors where scientists try to investigate and exploit the benefits of risk management. Research is focused, but not limited, to the fields of human errors, occupational safety (Reason, 1990), supply chain management (Ponis et al., 2006) and quite recently to project risk management (Kirytopoulos et al., 2001; Leopoulos et al., 2003).

The paper is focusing on risks that might affect an investment, thus putting in jeopardy the capital invested by a company or an individual. In nearly any type of investment, risks are most of the times regarded to be external. External risks are those that the probability of their occurrence cannot be affected by the investor (Leopoulos and Kirytopoulos 2004). In other words, the investor knows that risks might happen but cannot do anything to change the probability of their occurrence. Pike (1986) has modelled the investment decision through the use of statistics. He adjusted a normal distribution on each variable of the NPV function affected by risks and by using statistics calculations he reached an NPV outcome described not just as a single value but a normal distribution, thus providing valuable information to the investor.

The first part of the paper forms the definition of the problem, analyses and presents the conceptual model. The model aims at providing an alternative and innovative approach for investment appraisal, by applying optimisation methods (Genetic Algorithms (GA)) while simultaneously considering investment risk.

In order to highlight the characteristics of the conceptual model, as well as to describe the use of the method, a Case Study is presented at the second part of the paper. The Case Study concerns an investment in a tri-generation power plant (electricity, heat and cooling), fuelled by renewable energy sources, namely, several types of biomass. This particular type of example was chosen on the one hand due to the increased interest on renewable energy sources following the recent energy crisis and the respective extreme oil prices and on the other hand because this specific case possesses all the characteristics that make it an ideal candidate for applying the proposed conceptual model.

2 Method description

2.1 Problem formulation

The fundamental scope of the problem is to assess and select an investment project by optimising the investor's choices while in parallel considering the risk correlated with these choices. Among various economic criteria, NPV is selected, without loss of generality, as the one most commonly used.

The NPV of an investment depends on the values of a set of variables, which are either defined by the investor's decisions, or are fully determined by external to the decision system factors. A potential investor's aim should be therefore twofold: On the one hand, the best possible, in terms of NPV, investment should be selected by optimising the investor's decisions. On the other hand, specific knowledge is needed on the volatility of the external factors and on how this volatility affects investor's decisions, in regard to his risk policy.

It is therefore apparent that the investor's defined parameters and the external variables should be considered separately. The paper presents an integrative approach, which attempts first to compute the NPV in a way that heuristically optimises the decision-making process by shaping the best feasible investment and second, reliably quantifying its implied risk.

2.2 Conceptual model presentation

As stated above, NPV is a function composed of investor defined parameters, as well as external variables. In mathematical terms, the NPV function is described as

$$\mathrm{NPV} = f(\mathbf{P}, \mathbf{V}) = f(P_1, \ldots, P_n, V_1, \ldots, V_m) \quad (1)$$

where

$\mathbf{P} = [P_i]$, $i = 1, \ldots, n$, parameters that their value is defined by the investor
$\mathbf{V} = [V_j]$, $j = 1, \ldots, m$, variables that their value is an outcome of external factors.

The common approach of modelling external variables is to assume a single value for each one of them (Savvides, 1994). However, the deterministic consideration of a probabilistic reality is by definition erroneous. In this paper, a different approach is proposed. Let

$$V_j \sim X_j\,(\mathrm{mean}_j, \mathrm{std}_j) \quad (2)$$

that is, the variable V_j follows a statistical distribution X_j with known characteristics (as appropriate for each statistical distribution). Hence, the implied volatility of each external factor is not omitted, but modelled through a statistical distribution. The distributions' characteristics are expert-defined.

The first step of the approach is to assume that each external variable is equal to its most probable value, which, eventually, leads to a certain scenario, in this paper called probable $\mathbf{V} = \mathbf{V}_{\mathbf{probable}}$. Having determined the value of $\mathbf{V}$, the only thing that remains is to decide for the value of $\mathbf{P}$. The occurring optimisation problem $\mathrm{NPV} = f(\mathbf{P}, \mathbf{V}_{\mathbf{probable}}) = \max$ is solved via use of GA and returns as a solution the optimum vector $\mathbf{P}_{\mathbf{GA}}$.

$$\mathrm{NPV} = f(\mathbf{P}, \mathbf{V}_{\mathbf{probable}}) = \max \xrightarrow{GA} \mathbf{P} = \mathbf{P}_{\mathbf{GA}}. \quad (3)$$

GA comprise an optimisation and search technique based on the principles of genetics and natural selection. A GA allows a population composed of many individuals to evolve under specified selection rules to a state that maximises the selected criteria. GAs implement the biological concepts of genetic re-combination, genetic mutation and natural selection (Haupt and Haupt, 2004).

As the name implies, GAs are analogous to the biological concept of a gene, the basic building block for individual traits of organisms, and are combined in chromosomes to form the total, inherited characteristics of an individual. Pursuing the biological analogy further, genes and chromosomes change from one generation to the next so successor generations are never exactly the same as their predecessors. Recalling the "survival of the fittest" theory, alterations in genes and in the composition of chromosomes lead to the creation of individuals with new characteristics. Some of these characteristics are favourable in a given environment and others may be less favourable or even deadly. Consequently, populations find individuals with favourable genes and chromosomal combinations increasing in number and those with less favourable genes and chromosomal combinations decreasing in number. Over time, populations of organisms begin to find an optimal niche in the environment in which the gene pool is well balanced and effective or else the population begins declining (Grupe and Jooste, 2004).

Some of the advantages of GA include that it optimises even non-continuous and non-differentiable functions, with continuous or discrete variables, it does not require derivative information, it simultaneously searches from a wide sampling of the cost surface and it deals with a large number of variables. Even more, GA may succeed in finding the global optimum, due to the fact that the method evaluates simultaneously a large population instead of a single point for most non-heuristic optimisation methods.

These advantages are intriguing and produce stunning results when traditional optimisation approaches fall miserably (Haupt and Haupt, 2004).

The solution accomplished by the use of GA, $\mathbf{P_{GA}}$, provides the optimum values for parameters defined by the investor, which define the optimum investment decision, provided that the external factors will have values equal to their most probable value.

However, NPV $= f(\mathbf{P_{GA}}, \mathbf{V_{probable}})$ is just a point-estimate of the final NPV that will occur under realisation of the optimum investment scenario. But the $\mathbf{V_{probable}}$ is just a case among many that can happen, since V_j values are external to the decision system parameters. The method presented here proposes a more advanced approach by implementing a probabilistic NPV calculation, which is accomplished by the use of MCS.

MCS constitutes a stochastic statistical methodology of quantitative solution and risk assessment for non-deterministic problems, using a pseudo-population of randomly produced alternative scenarios from prescribed statistical distributions. The MCS suggests the modelling of the range of possible values for each input variable and the following reproduction of an efficient number of scenarios, so that the depiction of the respectively large number of results in a density function diagram could attribute in a reliable manner the needed distribution of the output variable, showing in parallel the possibility of occurrence for each value and marking out extreme or probable results (Vose, 2000).

A MCS is performed for $k = 1, \ldots, q$ iterations, where q is typically larger than 1000, by picking randomly values from the statistical distributions X_j, such that, for each j, k, occurs a $V_{jk} \sim X_j$ (mean$_j$, std$_j$). Each resultant vector $\mathbf{V}_k$ is used for computing a value of

$$\mathrm{NPV}_k = f(\mathbf{P_{GA}}, \mathbf{V}_k). \tag{4}$$

The NPV is finally calculated not as a single value but as a probability density function, which is based on optimum investor's decisions and represents the quantified volatility of external parameters.

The alternative representation of the q values of NPV derived from the MCS in a cumulative probability diagram also provides valuable information. The confidence level at the point of zero NPV could serve as a useful measure of risk. Moreover, the value of NPV at a selected confidence level provides a strong decision tool, which can serve as a decision criterion for the final acceptance of the project.

3 Case study

3.1 Case description

The Case Study concerns an investment analysis for a tri-generation power plant, given the demand of a specific customer for heat and cooling. Heat and cooling demand profiles are available for a specific residential sector of 500 customers in Greece. The revenues of the power plant under consideration will stem from selling electricity to the grid as well as heat and cooling to the customers via a district heating network that will be constructed. The price of heat is assumed to be a fixed percentage of the cost of heat obtained by using oil (as this is the common practice) whereas the price of cooling is a fixed percentage of the cost of cooling obtained by electrical compression chillers.

The power plant will consist of a base-load co-generation module and a biomass boiler for peak-load heat production. There are four biomass types available in the region, therefore all of them are considered as potential fuel sources for the plant. The multi-biomass approach has been used, due to the significant cost reductions that can be obtained at the logistics and warehousing stages of the biomass supply chain, compared to the single-biomass approach (Rentizelas et al., 2005). The basic characteristics of the biomass sources are presented in Table 1.

Table 1 Biomass characteristics

Biomass type	*Type 1*	*Type 2*	*Type 3*	*Type 4*
Price (€/kg)	0.05	0.01	0.03	0.01
Heating Value (KJ/kg wet)	10000	10500	12000	10200
Biomass availability (months)	September–October	February	February–March	May
Yield (tons/hectare)	2	4	5	3.5

The parameters included in the GA optimisation are:

- capacity rates for the base-load co-generation plant as well as the peak-load heat boiler
- amounts of each biomass source to be procured every year
- initial biomass inventory at the beginning of every year of operation.

The objective function (equation leading to the NPV value) of the optimisation problem is the NPV of the investment, during its operational lifetime. Investment considered includes the power plant, the supply chain of the biomass, the construction of the District Heating and Cooling network and connection to the customers, as well as the electricity transmission line and connection to the grid. The financial data used for the NPV calculation are presented in Table 2.

Table 2 Financial data

Interest rate (most probable value)	6%
Inflation rate (most probable value)	3%
Investment lifetime (fixed value)	15 years
Public Subsidy (most probable value)	40%

The specific case is highly suitable for optimisation with GA, as the objective function is non-linear and non-continuous, due to economies of scale of the biomass co-generation plant and analytical modelling of the biomass logistics network. Furthermore, several parameters may take a very wide range of values; therefore MCS is used to depict the effect that they may have on the NPV value. For example, oil price has undergone significant fluctuations lately, and has a significant dual effect on the NPV value, as, on the one hand it affects revenues from heat sales and on the other hand it affects reversely logistics cost for transporting and handling biomass. The characteristics of distributions X_j for all parameters V_j have been derived by experts and, the most important, are presented in Table 3. The third column of the table refers to $\mathbf{V_{probable}}$, which gives the values that have been used in the GA optimisation model.

Table 3 Characteristics of parameters' distributions

Parameter	*Distribution*	*Probable value*	*Standard deviation*	*Minimum value*	*Maximum value*
Income from electricity (€/kwh)	Triangular	0.06611	–	0.05	0.1
Income from electric power reimbursement (euro/kw/month)	Normal	1.7	0.2	1	–
Income from cool (euro/Kwh)	Triangular	0.036	–	0.03	0.05
Public subsidy on investment (%)	Uniform	40%	–	30%	50%
Total CHP efficiency rate	Triangular	0.85	–	0.75	0.88
Thermal CHP efficiency rate	Triangular	0.59	–	0.5	0.65
Thermal boiler efficiency rate	Triangular	0.8	–	0.7	0.85
Chiller efficiency	Normal	0.7	0.1	0.65	0.9
Inflation (%)	Uniform	0.03	–	0.02	0.05
Compound interest rate (%)	Uniform	0.06	–	0.05	0.08
Investment cost for CHP plant – reference case (€/kWel)	Normal	2000	100	1500	3000
Investment cost for Boiler – reference case (€/kWth)	Uniform	200	10	160	230
Diesel cost (€/lt)	Uniform	0.6	–	0.45	1
Biomass price – type I (€/ton wet)	Uniform	50	–	40	80
Biomass price – type II (€/ton wet)	Uniform	10	–	5	40
Biomass type – type III (€/ton wet)	Uniform	30	–	15	60
Biomass price – type IV (€/ton wet)	Uniform	10	–	5	30

It can be seen from Table 3 that different distribution types have been chosen to simulate the volatility concerning the values of various variables. For example, most of the variables that relate to construction work have been simulated with triangular distributions, as this is common practice in construction project management (Kirytopoulos et al., 2001). Other variables that have a certain value today, but are expected to follow a particular trend in the future (e.g., increasing electricity prices) are simulated also by triangular distributions, with positive or negative skewness. Financial figures that may fluctuate in an unpredictable manner have been simulated by uniform distributions, with logical upper and lower bounds. The adoption of MCS gets over the limitations of Pike's (1986) model that demands the use of the Central Limit Theorem. Thus, the limitation of using the same type of distribution to describe all the parameters included in the NPV value is omitted when the MCS is used.

3.2 Results

The optimum solution for the set of parameters P obtained by GA optimisation ($\mathbf{P_{GA}}$) is presented in Table 4. The optimum investment decision arises as clearly defined in terms of plant capacity, initial inventory and annual amount per type of biomass. The proposed approach provides a single optimum value for all the parameters that are determined by investor's choices.

Table 4 Optimised variables

Variables	*Value*
Thermal power capacity of CHP plant (kW)	4,388.56
Thermal power capacity of peak – load boiler (kW)	2,584.71
Total annual amount of biomass – type I (tons/year)	815.13
Total annual amount of biomass – type II (tons/year)	7,544.68
Total annual amount of biomass – type III (tons/year)	749.58
Total annual amount of biomass – type IV (tons/year)	7,004.80
Initial biomass inventory (m^3)	22,877.87

Following the adoption of these optimum prices, a MCS is performed to reveal the implied volatility that characterises the rest of the variables (Table 3). The characteristics of the NPV distribution that occurred through the 10,000 iterations of the simulation are presented in Table 5 that follows.

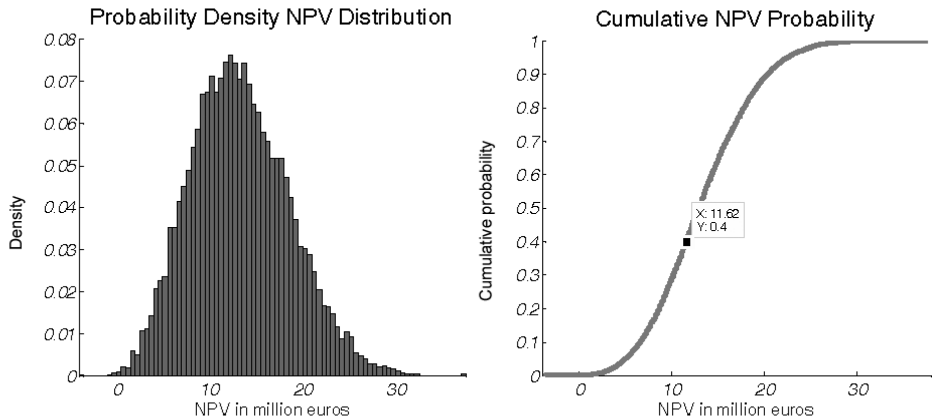
Table 5 NPV distribution characteristics

Minimum (€ × 10^6)	–3.50	Mode (€ × 10^6)	12.09
Maximum (€ × 10^6)	37.47	Left X (€ × 10^6)	5.08
Mean (€ × 10^6)	13.28	Left P	5%
Median (€ × 10^6)	12.95	Right X (€ × 10^6)	22.54
Std Dev.	5.31	Right P	95%
Variance	41.5	Diff X (€ × 10^6)	17.46
Skewness	0.327	Diff P	90%
Kurtosis	2.931		

An adequate amount of information is now available to the decision maker, in order to appraise, in his own customised way, the risk that is implied in the optimum investment. For example, the 'Left P' value indicates that there is a 95% possibility that the NPV of the proposed investment would exceed 5.08 million €.

The probability density function diagram of the simulated scenarios is presented in the left part of Figure 1. The diagram reveals in a visual way the possibility of occurrence of each NPV, and additionally marks out the most probable or extreme results. It also comprises an advanced but convenient view for the comparative assessment of two or more projects. The cumulative graph of the NPV distribution, which stands also as a very utile tool, is provided in the right part of Figure 1. It reveals straightway that the likelihood of a deficit is practically zero. Likewise, it provides the minimum value of the NPV for a given confidence level. For example, as it is pointed on the diagram, it states that there is only 40% possibility that the project's NPV will be under 11.62 million €.

Figure 1 NPV probability density distribution and cumulative probability distribution for optimum investment



The investor has now a sufficient amount of information in order to estimate a variety of customisable risk indexes and define in a reliable way the degree of project's attractiveness. For instance, the investor may find out what is the probability that the NPV will be negative, even if the investor-defined parameters have been optimally selected with the use of a scientific method. Thus the integrative approach presented here will come up with a risk analysis (Figure 1) of an already optimum decision and the investor will make the final decision based on his/her risk profile (risk averse/neutral/seeker).

4 Conclusion and future work

The paper proposes an innovative integrative approach to investment appraisal. The factors that affect the NPV of the investment under consideration are grouped into two sets. The first one contains the parameters that are determined by the investor's choices, such as the power plant capacity. This set substantially designates the investment's specifications. The other set is comprised of variables that their values are shaped by external to the investor decisions, such as the oil price. This type of variables is mainly responsible for the risk that is embodied in the investment.

The approach proposed in this paper suggests that the parameters should be calculated by the use of a GA, while the volatility of the variables will be modelled, afterwards, though the use of a MCS. The probabilistic modelling of the volatility of external variables is achieved through the use of expert-defined statistical distributions. The most probable values of these variables are adopted, in order to perform an optimisation of the selected economic criterion, which hereby is NPV, without loss of generality. The optimisation performed using the advanced GA technique, results in the best selection of the investor's defined parameters. The optimum investment specifications (indicated by the GA) are further used to perform a MCS for the external variable. The simulation results are finally used to form a probability density function or cumulative probability diagram of the criterion's value distribution. Hence, the decision maker is able to define the optimum investment and assess its risk in a reliable, convenient and customisable way.

The approach was demonstrated through a case study of an investment regarding a tri-generation power plant. The various external variables were stochastically modelled, and their most probable values were used to perform a GA optimisation for the NPV, through which the optimum power plant's configuration and operational characteristics emerged. The MCS that followed revealed the latent distribution of the NPV, and thus enabled the decision maker to assess his expected returns in a reliable quantitative way. The investment was finally fully determined, in terms of its design specifications, as well as the probability distribution of its expected NPV.

The proposed approach provides an integrated framework to investment appraisal. However, there is more to be done. Our current effort does not cope with some limitations implied in the GA optimisation. The evolutionary technique is very efficient in pointing the area of global optimum, but sometimes fails in determining its exact value. Moreover, the convergence speed at the late stages of optimisation process is slow. Additionally, computational time may be significant in some cases, as a large number of function evaluations (equal to the population size) is required at each iteration, as opposed to single evaluation of traditional gradient-based optimisation methods. The research team intends to continue improving the optimisation method, by taking into consideration these aspects. Our future work will also focus on proposing coherent customisable measures that quantify the NPV distribution in a reliable but simple to comprehend and easy to communicate way.

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